Further Information (FI) for Phys. Rev. Lett. submission *LA11843 Zhao*

PAPER TITLE: Internal group dynamics prolong a losing battle

AUTHORS: Zhenyuan Zhao, Juan Camilo Bohorquez, Alex Dixon and Neil F. Johnson

Contents:

We note that an unpublished e-print by us, available from $\mathrm{http://lanl.arxiv.org/pdf/physics/0605035v1}$, presents a preliminary empirical analysis of the distribution of casualties per encounter in human conflicts. While having nothing to do with the duration of conflicts, and hence nothing to do with the content of the present paper, the good agreement obtained provides some independent support for the dynamical clustering mechanism in our model.

For the sake of simplicity of presentation, we will continue to cast the discussion in the language of human wars – however we note that our results are potentially applicable to a wide range of other 'battle' scenarios involving the attrition between two populations (e.g. immunological responses, public health battles against latent disease, and even biochemical interactions, as mentioned in the main paper).

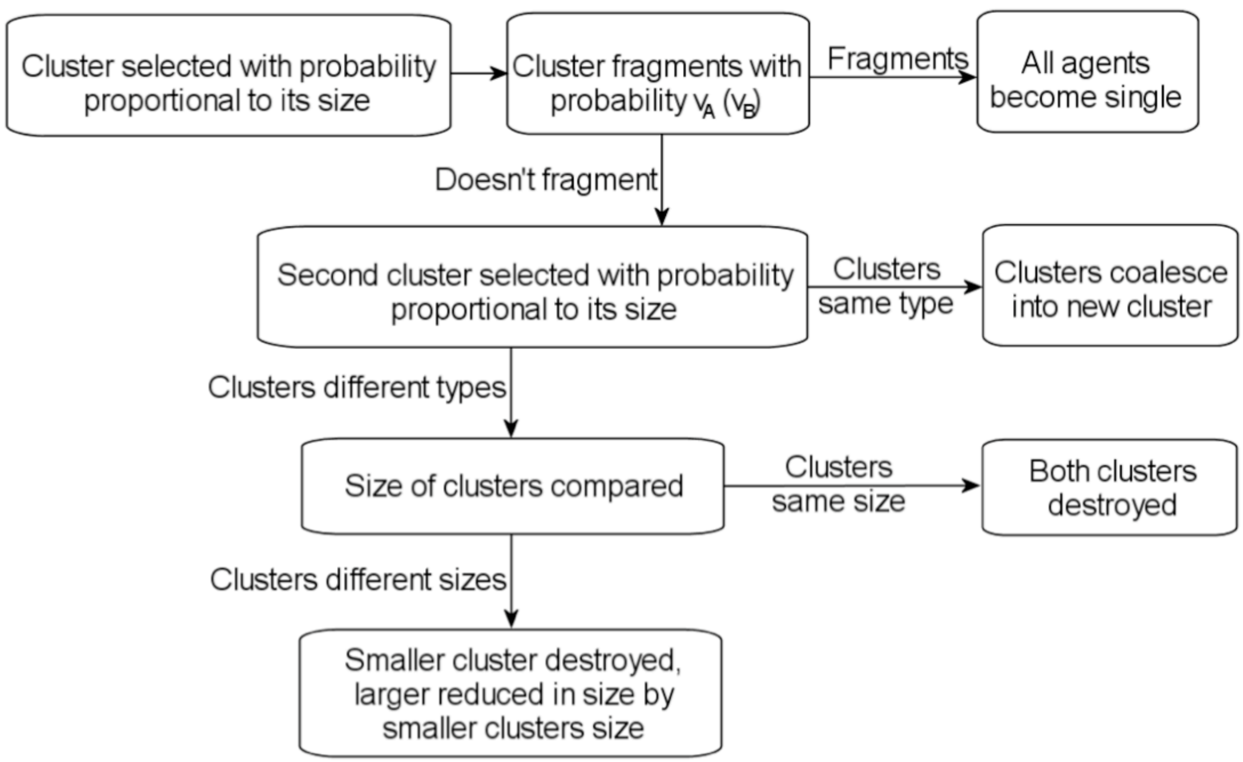


Figure FI 1: Flowchart of our basic model, prior to imposing any variations. This is the model which gives the results in Figs. 1 and 2 of the main paper, and generates the analytic result shown in Eq. (2)

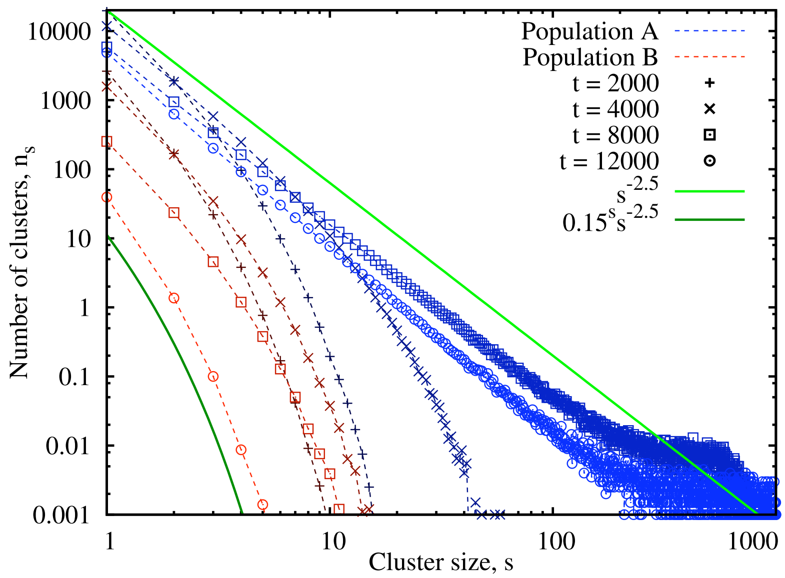


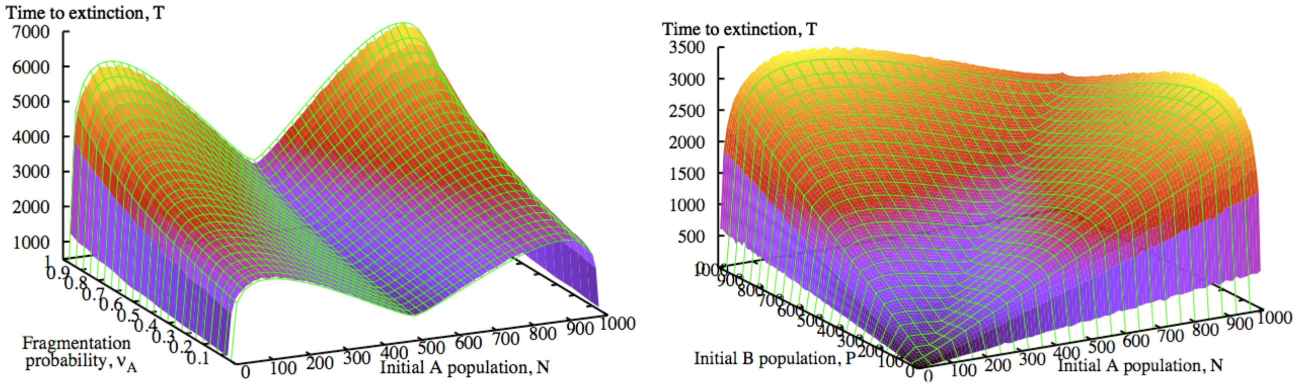
Figure FI 2: Distribution of clusters in population A of size N, and population B of size P, for various times during the war of attrition between A and B. The results correspond to our basic model. The A distribution has been rescaled by a factor of 5 for clarity. Initial conditions correspond to $N = 6000, P = 4000, \nu_A = \nu_B = 0.01$, with all clusters of size 1 (i.e. fragmented). The distribution is an average of 10^4 simulations.

1 Our basic model of conflict between populations A and B

Our model of conflict between populations A and B is presented in the main paper in its most basic form. Many generalizations are of course possible – however, the basic form is specifically constructed so as to feature dynamical clustering within a given population in a simple yet realistic way. The objective is to be able to understand the effect that such dynamical clustering will have on the survival time of a given population – or equivalently, its time to extinction, or the duration of the underlying war of attrition. Our model considers two populations of 'agents' engaged in a 'war', though more than two populations could be included. Indeed, in the main paper we do consider the effect of having a third population acting as blockers. As time evolves, the agents in either (or both) populations group together with their own kind to form clusters.

Consider two populations: A of total size N and B of total size P. The first population A has n_s clusters of size s and $\sum sn_s = N$ The second population B has p_s clusters of size s and $\sum sp_s = P$. It seems reasonable to expect that the respective clusters will evolve in time within a given war. Indeed, one can envisage that clusters will occasionally either break up into smaller groups or join together to form larger ones: for example, the opposing population A may be applying pressure in terms of searching for hidden clusters of B. Hence the clusters of B might either decide, or be forced, to break up in order to move more quickly, or in order to lose themselves in the environment. Our goal is to capture, in a simple way, these basic ideas of 'war' between A and B, such that clusters over time will fragment or coalesce with other clusters from their own population, or engage (i.e. fight) with clusters from the other population.

A flowchart of our basic model is contained in Fig. FI 1 of this FI document. In the limit of a single population, and in the absence of attrition, the model resembles closely the Eguiluz-Zimmerman model (see V.M. Eguiluz and M.G. Zimmerman, *Physical Review Letters* **85**, 5659 (2000)). As shown in Figure 1, at each timestep a cluster is picked from the total population, $N + P$, with probability proportional to its size . A justification for choosing a cluster with a probability which is proportional to its size, is as follows: a cluster with more members has more chances of initiating an event. In particular, it will have a higher chance of being more actively sought by the opposing population and hence may need to disband more readily, and/or it may run across members of the opposing population more often, and hence a higher chance of being involved in clashes or reuniting with other clusters of the same population. These rules form the heart of our basic model, as shown in Fig. FI 1. In addition to human conflict, this makes sense in a non-human scenario of two populations of cells or viruses and antibodies, since larger clusters will generally be bigger and hence more likely to collide or meet other clusters. This cluster then fragments with a fragmentation probability which, in general, is dependent on its population type, ν_A or ν_B. If the cluster does not fragment then a second cluster is selected from the total population, with probability proportional to its size. If the two clusters are of the same type (both A or both B) they coalesce. In the context



(a) Total population $N + P = 1000$, $\nu_B = 0.3$

(b) $\nu_A = 0.1$, $\nu_B = 0.3$

Figure FI 3: Time to extinction (T) in our basic model as a function of initial populations (N, P) and fragmentation probability (ν_A), from numerical simulations (surface) and the analytic theory (lines). Each data point is an average of 10^3 simulations.

of human conflict, the justification for choosing a cluster for coalescence with a probability which is proportional to its size, is as follows: it is presumably risky to combine clusters, since it must involve at least one message passing between the two groups in order to coordinate their actions. Hence it becomes increasingly less worthwhile to combine clusters as they get smaller. This again makes sense in a non-human scenario of two populations of cells or viruses and antibodies, since larger clusters will generally be bigger and hence more likely to collide or meet other clusters. If the two clusters are of different types (one A and one B) they interact. In the context of attrition, it makes sense to invoke a rule such as the following: In an interaction the smaller cluster is destroyed and the larger cluster is reduced in size by an amount equal to the smaller clusters size. If they are both of the same size (but opposite type) then both clusters are destroyed. In this way in any given interaction both populations lose the same amount of agents, which are removed from the model.

The result of this model is that populations A and B will, if allowed to coalesce and fragment in the above way, form broad distributions of cluster sizes. We assume that population B is an insurgent or guerilla group, and that a given cluster of B will produce casualties proportional to its size. The distribution of casualties will therefore be proportional to the distribution of clusters of population B. In Fig. FI 2, we show the distribution of cluster sizes of population B as a function of time. As can be seen, the distribution closely resembles a power-law, or at least a cut-off power-law, with slope of magnitude around 2.5. As shown in Fig. FI 3, this is remarkably similar to the behavior observed in the casualty distribution for the independent wars in Iraq and Colombia. In particular, taking a sliding time-window through the data in each case, the power-law slope stays remarkably close to 2.5. This value has even persisted throughout the recent changes in Iraq post-surge.

1.1 Analytic derivation of time to extinction

The time taken for one population to be destroyed (the time to extinction) is derived below. In our basic model, it is the smallest population that disappears first. The time at which this occurs is the duration of the war. Note that even if the war ends after a given fraction of the initial population is removed, the same qualitative results will still emerge since the theory is invariant under an overall change of scale. We also note that the members of the the two populations may not always be killed, they just disappear from the fighting force – hence the ‘attrition’ of $N_A(t)$ and $N_B(t)$ in the real wars of Fig. 1 of the main paper, may also be interpreted as a loss of strength or will-power as opposed to actual casualties.

We allow the simulations to start with initially fragmented clusters, in order to incorporate the initial procedure of building up larger clusters. In all the simulations, this initial build-up was very fast. As can be seen from Fig. 2 in the main paper, and Figs. FI 4, 5 and 6 in the FI, our analytic theory is in excellent agreement with the numerical simulations, even though it only captures the steps in the simulation in an average way. This suggests that the results in the main paper are

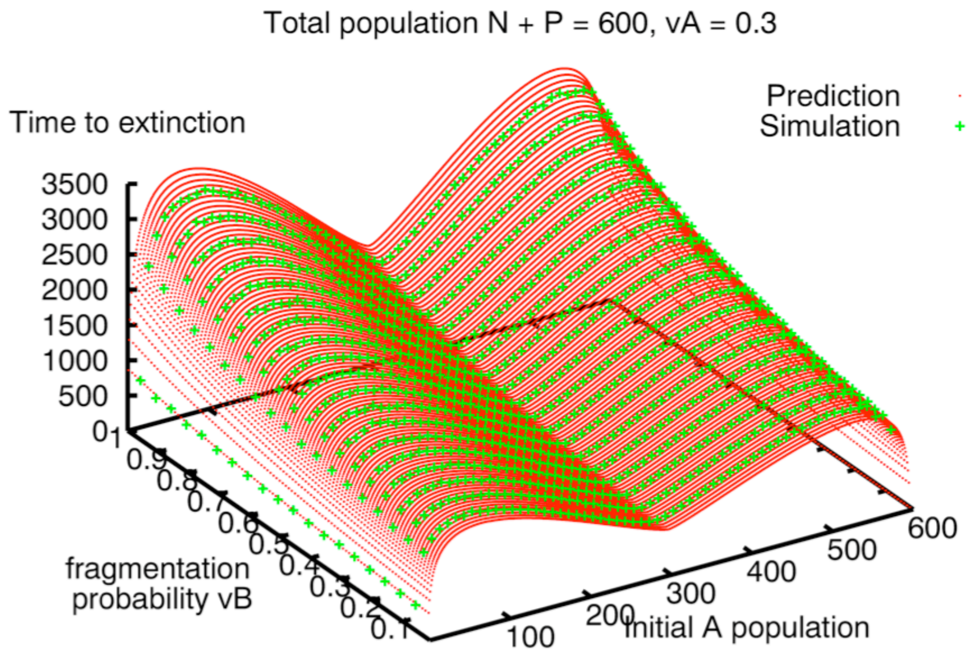


Figure FI 4: Time to extinction (T) in our model as a function of the initial A population (N) and fragmentation probability of B (ν_B), from numerical simulations (surface) and our analytic theory Eq. (1) (lines). Here the total initial population $N + P = 600$ and $\nu_A = 0.3$

very general, and do not depend on the precise parameter value choices for the numerical simulation. In other words, the main result of the present work are robust.

Our theory is as follows. The probability Q_{AB} that any cluster of population A is selected and interacts with one of population B is the sum of the probabilities for an A cluster of size s to interact with any B cluster, $q_{AB}(s)$. The first factor in $q_{AB}(s)$ is the probability for a cluster of type A and size s to be selected, the second the probability for this cluster not to fragment and the third factor is the probability for any cluster of type B to then be selected.

$$Q_{AB} = \sum_s q_{AB}(s) = \sum_s \frac{sn_s}{N+P}(1-\nu_A)\frac{\sum_r rp_r}{N+P}$$

$$Q_{AB} = \frac{NP}{(N+P)^2}(1-\nu_A)$$

Using the fact that $\sum sn_s = N$, $\sum rp_r = P$. The probability Q_{BA} of selecting a B cluster and it interacting with an A is given by a similar expression, with ν_A replaced with ν_B. After an interaction, each population A and B is reduced by an amount equal to the size of the interaction (which is the size of the smallest cluster in the interaction). Introducing an average interaction size c, the populations will become $N = N_0 - c$, $P = P_0 - c$. After i interactions the populations will be $N = N_0 - ic$, $P = P_0 - ic$, where N_0 and P_0 are the initial populations. Therefore the probability for an interaction between A and B after i previous interactions is

$$Q(i) = Q_{AB} + Q_{BA} = \frac{(N_0 - ic)(P_0 - ic)}{(N_0 + P_0 - 2ic)^2}(2 - \nu_A - \nu_B)$$

To reduce N (and P) by c takes $1/Q$ timesteps on average. The total time to reduce one population to 0 is the sum of the timesteps required for each killing interaction, until there are no agents left in the population. Taking P to be the smaller population, it will require P_0/c interactions to destroy it, so the final interaction will happen after $P_0/c - 1$ previous

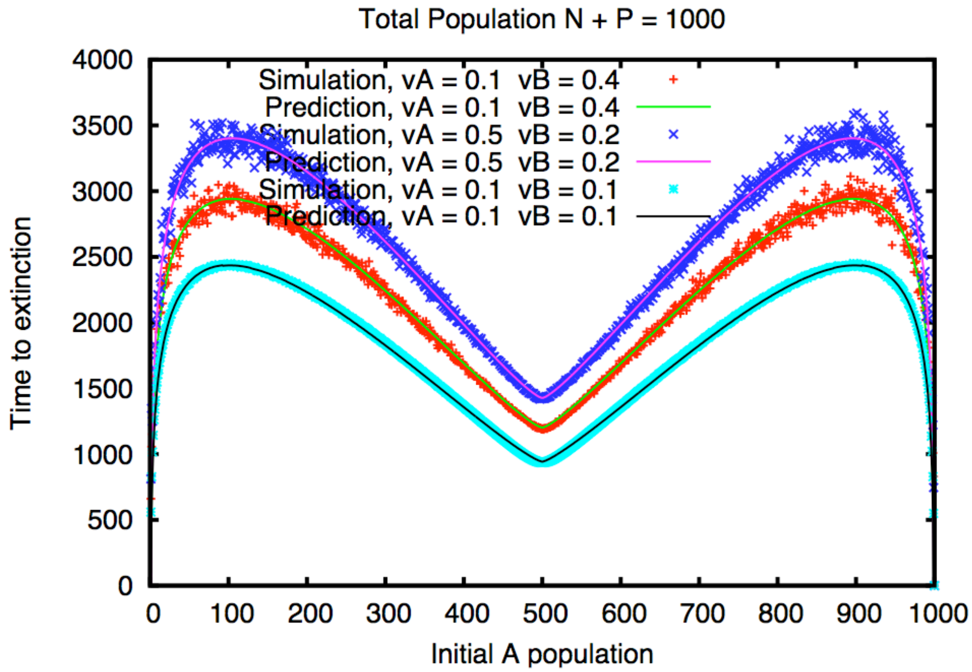


Figure FI 5: Time to extinction (T) in our model as a function of the initial A population (N), for different values of the fragmentation probabilities. Numerical simulations and our analytic theory Eq. (1). Here the total initial population $N + P = 1000$.

interactions.

$$T = \sum_{i=0}^{\frac{P_0}{c}-1} \frac{1}{Q(i)}$$

$$T = \sum_{i=0}^{\frac{P_0}{c}-1} \frac{(N_0 + P_0 - 2ic)^2}{(N_0 - ic)(P_0 - ic)(2 - \nu_A - \nu_B)}$$

$$T = \frac{1}{(2 - \nu_A - \nu_B)} \times \left(4\frac{P_0}{c} + \sum_{i=0}^{\frac{P_0}{c}-1} \frac{(N_0 - P_0)}{P_0 - ic} + \frac{(P_0 - N_0)}{N_0 - ic} \right)$$

$$T = \frac{N_0 - P_0}{c(2 - \nu_A - \nu_B)} \left(\frac{4P_0}{N_0 - P_0} + \sum_{i=1}^{\frac{P_0}{c}} \frac{1}{i} - \sum_{\frac{N_0 - P_0}{c}+1}^{\frac{N_0}{c}} \frac{1}{i} \right)$$

Using a standard result that the harmonic series can be expressed as $\sum_1^n \frac{1}{i} = \gamma + \psi_0(n+1)$, where γ is the Euler-Mascheroni constant and ψ_0 the digamma function, and the fact $\sum_{a+1}^n = \sum_1^n - \sum_1^a$ allows us to express T as

$$T = \frac{N_0 - P_0}{c(2 - \nu_A - \nu_B)} \Bigg(\frac{4P_0}{N_0 - P_0} + \Big(\gamma + \psi_0\left(\tfrac{P_0}{c} + 1\right)\Big) - \Big(\psi_0\left(\tfrac{N_0}{c} + 1\right) - \psi_0\left(\tfrac{N_0 - P_0}{c} + 1\right)\Big)\Bigg) \qquad (1.1)$$

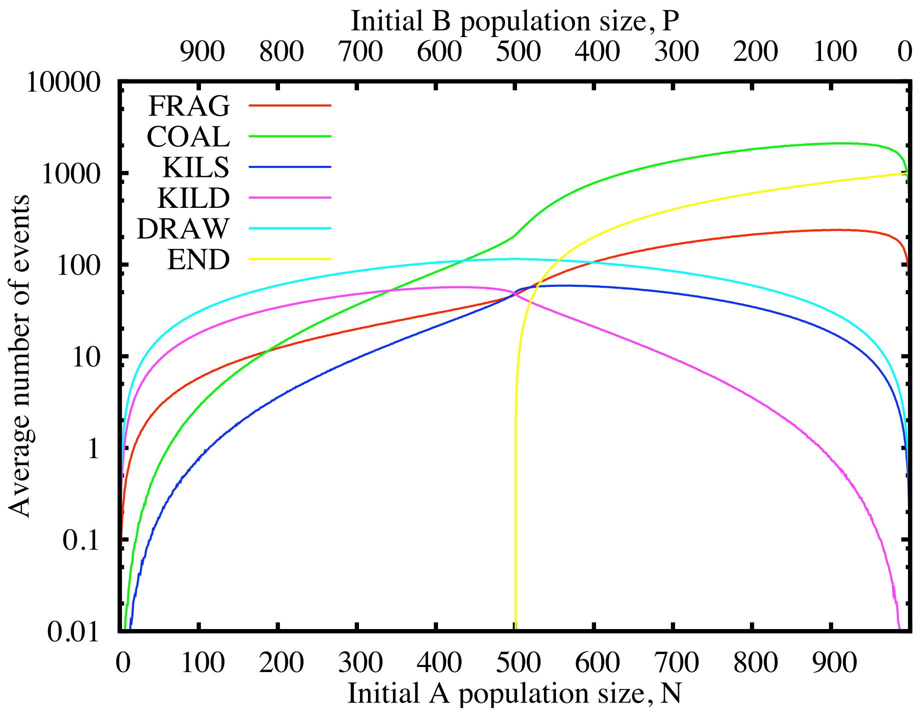


Figure FI 6: Average number of events in a decay as a function of relative initial populations in our basic model, with $N + P = 1000, \nu_A = \nu_B = 0.1$

This gives the time to extinction of a two population system in terms of the initial larger (N_0) and smaller (P_0) populations, their fragmentation probabilities (ν_A and ν_B) and the average size of a (fatal) interaction, c. The average interaction size depends on the cluster size distributions, hence on the initial populations and fragmentation probabilities. It cannot be calculated as the cluster distributions do not reach a steady state during decay, however the variation from 1 is found from numerical simulations to be small and approximately linear,

$$c = 1 + 0.2(P_0/N_0)(1 - \nu_B)(1 - \nu_A) \tag{1.2}$$

where P_0 is the smaller population size and N_0 the larger. There are two simple alternative approximations to c which can be invoked – that $c = 1$, or that c is given by $s_{\min}\frac{\alpha-1}{\alpha-2}$ which is the average size of a cluster for a power-law distribution with exponent α and a power-law onset at $s_{\min} = 1$. For $\alpha = 2.5$ and $s_{\min} = 1$, this latter approximation yields $c = 3$. It turns out that the agreement between the theory and numerical simulations is good for all these c values. In other words, the precise value of c is not important and hence we can simply use $c \sim 1$.

1.2 Numerical simulation results

Typical results from the numerical simulation are shown in Figure 2 in the main paper, and in Figs. FI 3, 4, and 5 of this FI document. The same qualitative nonlinear dependence of the duration on the population asymmetry, appears in all cases and is independent of the initial population sizes and fragmentation probabilities (ν_A, ν_B). It is robust. The actual numerical value of the duration time for a given asymmetry, does depend on all of these variables – but as shown in Figure 1 of the main paper, we are able to capture the same general pattern across a large number of new wars using a constant value for the fragmentation probabilities (ν_A, ν_B). This suggests that the model is indeed capturing the essence of these wars' extended durations, and that the model itself is robust.

The observation that time to extinction increases with fragmentation probability can be explained within the model as follows: Interactions between the populations only occur when a cluster is selected and doesn't fragment, and interactions are required to destroy agents. For a given ratio of N to P, the time to extinction also increases with total population $N + P$: More agents require more time to be destroyed.

Interestingly, the dependence on the ratio of N to P is nothing like we would intuitively expect. In any kind of fight, we would imagine a strong opponent would destroy a weaker one much more quickly than if the two sides were of comparable strength. However, both numerical simulations and the analytic theory show that the opposite is actually true; the larger the imbalance in strengths the longer the fight lasts. A population of 100 and 900 agents takes considerably longer to decay to extinction than two equal populations of 500. This surprising result can be understood by looking at the average number of 'events' that occur in a given decay as a function of initial A population (keeping total population constant, so B population $P = 1000 - N$), Fig. FI 6. The events are:

FRAG - Type A cluster selected and fragments
COAL - Type A cluster selected and coalesces
KILS - Opposite type clusters selected, A bigger
KILD - Opposite type clusters selected, B bigger
DRAW - Opposite type clusters selected, same size

and **END**, which is not an event but the final A population. The number of type A coalescences and fragmentations (inter population activity) increases rapidly as A becomes the majority population (note logarithmic scale). At the same time the number of destructive interactions (KILD, KILS, DRAW) between the populations decreases. As the probability for any cluster being selected is proportional to its size, the probability for any population being selected is proportional to its total size. For unequal populations there is a greatly increased probability for the larger population to self interact, the same population being selected twice, compared to equal populations where the probability for self and opposite interaction is the same. As destructive events only occur with opposite interactions, this results in an increase in the time between agents being destroyed in asymmetric populations. This is not offset by the decrease in time due to fewer agents to destroy, leading to a net increase in the time required for extinction.

2 Model robustness to changes in parameter values, and changes in microscopic rules

Here we list some variations to our basic model, and demonstrate that they retain the same qualitative behavior as reported in the main paper (i.e. Figs. 1 and 2).

2.0.1 Casualty variant: EFF model

Consider a variant of our model, which we refer to as EFF, in which we vary the way in which casualties occur in order to check that our main finding is robust to changes in such details. There are again two populations of agents, denoted type A and B, which can group together to form clusters within their populations. On each timestep a cluster is selected from the total population (A+B) with uniform probability, so each cluster has the same chance of being selected. A cluster is then selected out of the total population with probability proportional to its size, resulting in each agent having the same probability of being selected. The two clusters are then compared; if they are of the same type (A or B) they coalesce. If they are of different population type, then they interact in such a way that the smaller cluster loses half its agents before fragmentation occurs. The larger cluster loses an amount of agents equal to the number lost by the smaller cluster, but does not fragment.

Our numerical simulations show that the end state of the system involves the initially smaller population being completely destroyed, while the other population remains in a single cluster with size equal to the difference between the initial populations. This generalization still retains the main findings of our basic model (i.e. Figs. 1 and 2 of the main paper). As shown in Fig. FI 7, the time until extinction is much greater for an asymmetric population than for a more symmetric population.

2.0.2 Minority advantage variant

Our basic model is modified by changing the behavior in the case of a draw (that is, when two clusters of opposite species but the same size are selected). Previously both clusters were destroyed – but now, only the cluster belonging to the larger total population is destroyed. This could mimic a particular 'home advantage' for a cluster from the minority population B, when faced with an equal-sized cluster of the invading army A. Despite the difference in the rules as compared to the basic model, the graph of the time to extinction graph (Fig. FI 8) has a similar shape. This gives us further confidence that the results of the main paper are robust.

3 Details of the distribution of time-intervals between conflict events

The average reaction size c has small variation from 1 and from numerical simulations, and is approximately given by:

$$c = 1 + 0.2\frac{1-x}{1+x}(1-\nu)^2 \quad (3.1)$$

Taking a close look at the analytic expression for duration T, we see that it depends only on the initial population of A and B, $T \equiv T(N_A(0), N_B(0), \nu)$. Assuming $N_A(0) + N_B(0) = N$ is constant,

$$T \equiv T(x, \nu) \quad (3.2)$$

where $x = |N_A(0) - N_B(0)|/(N_A(0) + N_B(0))$ is the initial asymmetry. Taking the derivative of T with respect to x and setting it equal to zero, we get an expression for the asymmetry value x at which the duration T is a maximum. This gives $x_{max} \simeq 0.788$ *independent* of ν. Within the small range of asymmetry values x around x_{max}, this equation for T can be assumed to be separable:

$$T(x, \nu) \cong t_1(x)t_2(\nu) \ . \quad (3.3)$$

Substituting in x_{max}, we obtain $t_2(\nu) \sim a/(1-\nu)$ where a is a constant (see Fig. 9(b)). This analysis reveals that there are two factors determining the duration T: one originates from the asymmetry x (i.e. $t_1(x)$), while the other originates from the microscopic clustering dynamics within a given population, ν (i.e. $t_2(\nu)$).

4 Data sources for Figure 1 of the main manuscript

Now we turn to the data sources and methodology which were used to prepare Figure 1. We purposely used sources within the openly available literature, in order that any interested readers may either repeat or extend our analysis.

For each of the wars presented, we deduce a duration for the conflict and estimate the asymmetry in the strength of the forces fighting it. Our paper's main result is to show that the presence of dynamically evolving groups in the minority population can dramatically increase the total interaction (e.g. conflict) time – Figure 1 provides support for this finding from the field of human conflict. Human wars are highly complex and evolve in time in a complicated way – moreover, their precise beginning and end are often debatable and the cause of much discussion among historians. We want to avoid such details, and therefore measure the duration of the war starting from the first day of the month in which the war was reported to have started. This is an approximation, but one which does not introduce any systematic bias. Given that the wars shown last much longer than 30 days, this is a negligible effect. For those wars that are still ongoing, we measured the days up to the end of 2008, and marked them accordingly in Fig. FI 1 of our paper with an arrow pointing up. The more time which lapses between the end of 2008 and the present day, the more the various triangle datapoints will move upward.

In terms of estimating the asymmetry in the strengths of the populations (i.e. armies) A and B, there are various issues. First, reliable estimates are hard to find. Second, some wars may involve various types of insurgents. e.g. paramilitary and mercenary groups can oppose both forces or even change sides in the middle of a conflict. However, we believe that these effects are not too important for the wars that we showed. We also emphasize that, to our knowledge, there are no wars that one might have considered including, but which are not shown because they do not fit the curves. All the new wars that we could list and find numbers for, are shown – with the caveat that we chose wars that in some sense were 'stable' in that they weren't some simple, rapid uprising lasting 1-2 days. Instead, they involved many combatants and lasted sufficient time so as to be considered a 'war' in the historical literature.

Just as in the main paper, we measured the asymmetry as follows: $x = |N_A - N_B|/(N_A + N_B)$ where N_A and N_B are the estimated total peak sizes of the two opposing forces A and B. Because of the normalization in the denominator, the asymmetry x takes a value between -1 and 1 where 0 means there is no asymmetry between forces, and 1 (or -1) means that $N_B = 0$ (or N_A=0). Another issue with measuring asymmetry is the fact that forces vary through time. For experimental and practical reasons, we estimate the strength of the opposing forces at their peak. In a war of attrition, the peak corresponds to $t = 0$ since there is no additional recruitment at a later time. Even if numbers do actually change in time through recruitment, the results can still hold if the relative killing rates are suitably adjusted. Another way of seeing this, is that we are assuming that the asymmetry in strength is well-defined near the start of the war. We hope that any interested historians might offer additional information for us to refine this analysis. At the present time, all we can do is follow the procedure of deducing the numbers quoted in the literature, calculating the asymmetry, and plotting the duration for this asymmetry on Figure 1 of the main paper.

In terms of the theoretical curves in Fig. 1, we simply use the analytical expression for the duration in each case, by plugging in the parameter values $N_A(0) + N_B(0) = 1000$, $\nu_A = \nu_B = 0.6$, $a = b = 0.00263$ and $c = d = 0.000033$. These are chosen so that all curves pass very close to World War II since World War II arguably denotes the transition from 'old' to 'new' wars. Despite the crudeness of this exercise, it is remarkable that the results in Fig. 1 seem so clear in their support for a non-linear relation between asymmetry and duration.

We now go through each war in turn, and discuss the numbers which we used and their sources.

4.1 Afghanistan War

The Afghanistan War refers to the current conflict being waged in Afghanistan. It began with the invasion of Afghanistan by forces of the United States and the United Kingdom as retaliation against the Taliban regime following the 9/11 attacks on the Twin Towers in New York.[1] One could argue that the Afghanistan War had in fact been going on for far longer – this would actually improve the agreement with the theory by pushing the triangle higher in the vertical direction. However we avoid doing that here, since we want to stick with the commonly accepted definition of each war.

4.1.1 Estimated Duration

The war started on October 7, 2001, with the bombing of Kabul by US forces. The first news of the attack appeared on October 7 2001, on CNN. We took this day as the start of the Afghanistan War. Since there are still operations going on in Afghanistan[2], we take this war to be ongoing and therefore assign an upward arrow to it in Figure 1.

4.1.2 Estimated Asymmetry

We assume the two opposing forces operating in Afghanistan are the pro-Taliban insurgency (B), and the US-led coalition forces (A). The following sources give estimates of the number of fighters on both sides in Afghanistan:

- Pajhwok Afghan News reported 20,000 rebel combatants vs. 130,000 coalition forces including US and the International Security Assistance Force (ISAF) forces. '(In) Kabul, 50,000 Afghan National Army (ANA), 60,000 policemen, 35,000 ISAF soldiers and 8,000 American troops were fighting it out with insurgents to pave the ground for reconstruction'[3].
- NATO reports 47,000 troops, organized as the ISAF from different countries. [4]
- The Guardian estimated 7,700 UK troops were deployed to Afghanistan [5].
- The Toronto Times referred to 13,000 and 27,000 in May 2006 [6].
- The Heritage Foundations estimated between 2,000 to 4,000 fighters in 2005, mentioning that the number of fighters had grown since [7].

We therefore took an average estimate of 80,000 coalition forces and 16,000 pro-Taliban forces, giving an asymmetry of 0.66. We decided not to include the police force inside the coalition force.

4.2 Algerian Civil War

The Algerian Civil War refers to the period of civil unrest where Islamic groups fought the Algerian government for control.

4.2.1 Estimated Duration

We took December 1991 as the start of the war, when rebels launched their first attack [8]. Although there is still some reported violence [9], February 2002 seems to be a reasonable estimate of the end of the war. In 1998 the Armed Islamic Group (GIA) was condemned by the civil population and other Islamic groups such as Al-Qaeda for their use of indiscriminate massacres [10]. Finally in February 2002, Antar Zouabri, the leader of the GIA, was gunned down [11] thereby bringing an end to the violence that started in 1991. Although the Salafist Group for Preaching and Combat (GSPC) branched off from the falling GIA in 1998, the small numbers involved (100 to 200 fighters) and the differences from the GIA make us believe that it is more related to the uncoordinated violence in Africa due to various Islamic rebel groups, as opposed to the Algerian Civil War.

4.2.2 Estimated Asymmetry

We assume the two opposing forces that operated during the Algerian Civil War, were the different Islamic factions (B) and the government forces (A). The following sources give estimates of the number of combatants:

- The Algerian military was estimated at 140,000 in 1994, and 124,000 in 2001 [12].
- Al Jazeera estimates the Algerian military as having 136,000 active troops [13].
- Martinez also estimates the size of the Islamic groups to be around 40,000 in 1994 [12].
- Around 1994, the Islamic Salvation Army (AIS) (part of the belligerent force) was estimated at between 12,000 and 15,000 men [14].
- BBC reported that the GIA had around 300 fighters by 2003 following Antar's death [11].
- The total support for the Islamic Salvation Forces was estimated to be 100,000 in the article 'Anatomy of a Massacre'. [15].

We therefore estimate around 130,000 armed forces and 35,000 militants, yielding an asymmetry of 0.57. We deduced the 35,000 man-estimate for the militias from the fact that at least 7 armed groups operated in the peak of violence around 1997 [16] together with the facts of the AIS having around 15,000 men, the reference of 40,000 of men by Martinez, and the lack of details regarding how many of the 100,000 supporters were actual combatants.

4.3 Colombian Conflict

The Colombian conflict refers to the period of armed insurgency from 1984 to the present. Although some researchers state that the conflict has been ongoing since the independence from Spain[17], the organized insurrection began after 1980 following the Seventh Communist Conference when the FARC organized themselves into FARC-EP (i.e. the people's army) [18]. Similarly the M-19 organized itself in 1972 following unrest after the 1970 elections[17]. The ELN's first armed action occurred in 1965. Other guerrilla groups such as the EPL, emerged during the 60's and 70's. The final components of the present conflict appeared in 1980, and were mostly drug cartels.

4.3.1 Estimated Duration

We took 1984 as the approximate starting date of the modern Colombian conflict. Even though guerrillas had emerged prior to 1984, this prior existence seems more related to the general ongoing violence which had plagued Colombia for more than 100 years [17]. By contrast, 1984 appears to be a watershed period in terms of escalation for the following reasons. Modern insurgent techniques, and tactics were deployed after the seventh conference that took place in 1980. After 1982 there was a truce in which most actors participated [17], but which had then disintegrated by 1986. In 1985, the biggest armed action by M-19 occurred when they stormed the Colombian Palace of Justice. Recent investigations try to link this action to drug traffic[19]. Drug cartels are a vital part of the modern Colombian conflict, and therefore we believe that the present conflict took form while this truce was in place (1982-1986), since new alliances with illicit drug groups began to emerge in that period. We note that if we were to assume that the war started earlier, it validates our research even more since it yields an even more anomalous duration.

4.3.2 Estimated Asymmetry

We assume two forces are in operation in Colombia, the guerrillas (B) and the government forces (A). The guerrillas include FARC, ELN, M19, EPL and other smaller groups. This is fine, since the clusters in our model of insurgency are largely disconnected. For the government, we took the combination of the armed forces and the paramilitary forces that operated in the different areas, since they began under protection of the government [17]. The reports are as follows:

- In 2001 Marks estimated 145,000 men in the Colombian military around 1999, and 16500 FARC combatants[18].
- An estimate of 16,000 combatants in 2001, and 8,000 in 2008 was given by the BBC in February 2008 [20].
- Between 9,000 and 16,000 combatants in the FARC, according to globalsecurity.org [21]
- An online reference cites ELN with 3,000 to 5,000 militants[22], and the FARC as having 9,000 to 15,000 militants[23].
- The Naval Postgraduate School estimates around 3,000 ELN members in 2006 [24], and 15,000 FARC rebels [25], referencing in both cases the U.S. Department of State as a source.
- M-19 was estimated as having 1,500 to 2,000 men in 1985 [26].
- An estimate of 120,000 soldiers was made in 2001, during the peace talks in El Caguan [27].
- In 1988, it was estimated that the Colombian army had 80,000 active personnel, compromising 80% of the total personnel. [28].
- In2002, there was an estimate of 9,000 men in the United Self-Defense Forces of Colombia (AUC), the most organized paramilitary group in Colombia [29].
- The AUC strength was estimated as between 6,000 and 8,150 [30].

Using these numbers, we estimate an asymmetry of 0.75 between the insurgent factions over the Colombian military. This is based on an estimate of 17,000 insurgents (B) and 110,000 soldiers plus 10,000 paramilitaries (A).

4.4 El Salvador's Civil War

The Salvadorian Civil War refers to the period of civil unrest in El Salvador involving leftist guerrillas and the government.

4.4.1 Estimated Duration

The conflict began on October 15, 1979 when the Revolutionary Government Junta took control, removing Carlos Humberto Romero from power. However the Junta dissolved due to internal pressures, and the problems quickly erupted into a civil war. The violent escalation began with the assassination of Monsignor Romero on May 1980[31]. The conflict continued up to 1991 when a truce was declared in April. The conflict ended in January 1992, with the creation of a new constitution. We will take January 1992 as the end of the conflict, since during negotiations the military force of both parties affected the political negotiations. We assume this because the treaty ended in a new constitution, and not by the submission or defeat of one group [32].

4.4.2 Estimated Asymmetry

We assume that the two factions that confronted each other in El Salvador, were the communist guerrillas (B) and the government (A). The following sources give estimates of the strength of the armies.

- Frente Farabundo Mart para la Liberacin Nacional (FMLN) was estimated as having 6,000 to 7,000 combatants [33].
- Bernard Cloutier through his web page, estimates the Salvadoran army in 1991 (i.e. around the time of the truce) as having 60,000 members [34].
- In an article in 1989, the US Department of State assessed the number of guerrillas as having been around 12,000 in 1983 and 6,000-7,000 in 1988 [35].

Based on these numbers we estimate the asymmetry as 0.72, with 8,000 guerrilla fighters (B) against 50,000 soldiers (A).

4.5 Iraq War

We refer here to the ongoing war in Iraq, with US forces and their allies (A) fighting the insurgency (B).

4.5.1 Estimated Duration

The war began with the invasion of Iraq on March 20, 2003 [36] and is an ongoing conflict.

4.5.2 Estimated Asymmetry

We assume that two opposing forces are waging the war: the United States and its allies (population A) and the guerrilla factions who oppose them (population B). This is obviously an enormous simplification of the complex dynamics that are actually underway in the conflict. However even though the various guerrilla groups might occasionally fight among themselves, they have a common opposition to the occupation of Iraq and hence can be seen as a single population B who employ (most of) their resources in fighting the coalition forces A. We have the following sources as possible estimates of the number of people fighting in Iraq:

- The Brookings Institute published a report with the following numbers: 130,000 to 140,000 US troops deployed by March 2007 (p.5 of Ref. [37]); 79,000 Iraqi security forces and 50,000 US Coalition forces operating in Baghdad and the surroundings (p. 7 of Ref. [37]); 20,000 to 30,000 insurgents estimated in October 2006 (p. 26 of Ref. [37]); 180,000 total coalition troop strength in September 2007 (p. 27 of Ref. [37]); 359,700 security forces, including 194,200 policemen and 165,500 in the national guard (p. 34 of Ref. [37]).

- The Washington Post refers to 60,000 militia fighters according to non-disclosed intelligent officials in January 2007 [38]. They also refer to 165,000 troops deployed by December 2006.

- In 2006, the Washington Post gave an estimate of 1,300 foreign militias from Al-Qaeda, and an estimate of 20,000 to 30,000 militias [39].

- According to Global Security, ‘on August 3, 2006, Secretary of Defense Donald Rumsfeld testified that 133,000 US personnel were deployed in Iraq’ [40].

Given these numbers, we assign an asymmetry of 0.66 based on an estimate of 150,000 soldiers (A) and 30,000 militants (B).

4.6 Peruvian Internal Conflict

The Peruvian internal conflict refers to the period of confrontation between the communist guerrillas in Peru (population B) and its government (population A).

4.6.1 Estimated Duration

The war started in 1980 when the Peruvian government decided to hold elections after 12 years. However, the Shining Path group decided not to participate in the elections, and instead declared a protracted war on the government to take power of Peru. The first armed action of the Shining Path was to boycott these elections. Later on, other guerrilla groups emerged such as the Tpac Amaru Revolutionary Movement (MRTA).

On September 12, 1992 the leader of the Shining Path, Abimael Guzman was captured by government forces. Later in 1999 the Peruvian government captured Oscar Ramirez, the man who took Guzman’s place in the guerrilla command structure. With his capture, the number of Shining Path armed actions diminished abruptly. Similarly the government captured and killed the members of the other guerrilla groups in various operations. One of the most famous was the 1996 occupation of the Japan embassy in Peru. These operations ended with a government raid, where the hostages were liberated and all the MRTA members were killed.

From 2000 onward, there has been little presence of the guerrillas in Peru and they are not able to operate freely. Also their goal has changed from taking over the entire Peruvian state, to the liberation of all imprisoned guerrilla fighters. We will therefore take January 2000 as the date the war ended – at least in the way that it had been waged since 1980.

4.6.2 Estimated Asymmetry

Regarding the size of the militia groups, we found the following sources that relate to their strength:

- The New York Times reported in 1993 that the number of Shining Path had diminished to 3,000 fighters, ‘about half of its ranks’, since the capture of Guzman. This would give us a rough estimate of 6,000 fighters [41] .
- In Terrorism Focus, a publication of The Jamestown Foundation, we found an estimate that the Shining Path numbers had gone down from 10,000 to 500 by that year [42].
- Globalsecurity.org estimates that the Shining Path had around 10,000 combatants [43].
- A website based on the Country Studies Series by the Federal Research Division of the Library of Congress, states that the Shining Path had 3,000 to 4,000 members in 1992. The MRTA had between 750 and 1,000 members, and the Peruvian military had around 75,000 members (62% were conscripts)[44]

Based on these sources we estimate around 10,000 guerrillas (B) fighting 75,000 military forces (A), giving an asymmetry of 0.76.

4.7 Spanish Civil War

The Spanish Civil War refers to the conflict that took place in Spain between 1936 and 1939, between the Spanish Republic and Nationalist Spain.

4.7.1 Estimated Duration

The war is considered to have begun on July 17, 1936, when the nationalist-traditionalist rebellion took place [45]. The war officially ended on April 1, 1939, when the Nationalist army led by Francisco Franco, took control of the remaining strongholds of the republican forces and announced the end of the war on public radio.

4.7.2 Estimated Asymmetry

The following sources estimate the size of the armies:

- Thomas Hughes estimates that the Republican forces were around 450,000 (p. 628 of Ref. [46]) and the Nationalist forces were around 600,000 (p. 619 of Ref. [46]).

We will take this numbers as a suitable reference and hence estimate an asymmetry of 0.14.

4.8 Sierra Leone Civil War

We refer here to the civil war fought in Sierra Leone between the government and the revolutionary armies.

4.8.1 Estimated Duration

The war is thought to have begun on March 23, 1991, when the Revolutionary United Front (RUF) launched an attack on the government of Sierra Leone from Liberia. The war ended on January 18, 2002, when president Kabbah declared the end of the civil war after the truce set in 2001.

4.8.2 Estimated Asymmetry

There are few sources that estimate the sizes of these armies. Here are the ones that we found:

- The United Nations program for Disarmament, Demobilization and Reintegration (NCDDR) which was prepared in April 1998, aimed to disarm and reintegrate all 75,000 combatants in the conflict – based on the recommendations of the World Bank. It estimates ‘10,000 ex-Sierra Leone Armies (SLA)/Armed Forces Revolutionary Council (AFRC); 55,000 Civil Defense Forces (CDF); 7,000 RUF and 3,000 child combatants as well as 300 disabled’ [47].
- The United Nations Security Council estimated 15,000 RUF combatants[48].

- Mark Malan in his essay ‘Layered Response’ on the Sierra Leone conflict, estimated the AFRC as having 14,000 combatants. It also references the British Army’s Draft Military Reintegration Plan as estimating that from the 45,000 combatants that demobilized, 15,000 were from the CDF, 15,000 were RUF, 2,000 paramilitaries, 6,000 SLA and 7,000 ex-SLA/AFRC [49].

We assume that the two sides were as follows: on the government side, there were the SLA, paramilitaries and CDF. On the opposing side, there were the RUF and ex-SLA/AFRC forces. From the estimated numbers, we deduce an asymmetry of about 0.2, based on the government forces being around 42,000 combatants (20,000 CDF plus 20,000 SLA plus 2,000 paramilitaries), and the revolutionaries to be around 28,000 combatants (15,000 RFU plus 10,000 AFRC plus 3,000 child combatants). These are rough estimates, but the numbers suggest a reasonably symmetric conflict.

4.9 US Civil War

The US Civil War refers to the war waged between The Confederates and The Union between 1861 and 1865.

4.9.1 Estimated Duration

The war began on April 12, 1861, when Confederate troops bombarded Fort Sumter. The war ended on April 9, 1865, at Appomattox Court House when General Lee surrendered to the Union.

4.9.2 Estimated Asymmetry

There has been a lot of research dedicated to the American Civil War. The following sources estimate the strength of both armies:

- In an article about facts of the war, the New York Times estimates that the Union had around 2,772,000 men, and the Confederation around 1,500,000 men. [50].
- Another estimate from the University of Houston gives the Union 2.5 million to 2.75 million men and the Confederation 750 to 1.25 million men [51]

Given these numbers we estimate an asymmetry of approximately 0.39.

4.10 Vietnam War

The Vietnam War refers to the war fought between 1959 an 1975 in Vietnam, between communist factions (B) and the US support of the Republic of Vietnam (A).

4.10.1 Estimated Duration

The war began to gather strength from 1956. Following the separation of Vietnam by the Geneva Conference of 1954, South Vietnam (supported by the US) began a military campaign against the communist faction, calling them the Viet-Cong. North Vietnam was pressured to take a stronger stand on the unification of Vietnam. A low-level insurgency developed in south Vietnam targeting political figures, but soon escalated. Finally in January 1959, following pressure from the Diem police in South Vietnam, the North authorized the beginning of an ‘Armed Struggle’ effectively initiating the war. We will take January 1959 as the beginning of the war.

The war came to an end on April 30, 1975, when the last US marines in Vietnam territory, evacuated the embassy by helicopter. During the day, the Vietnam’s People Army (VPA) overcame all resistance and took control of key buildings and structures ending the struggle.

4.10.2 Estimated Asymmetry

There were two factions fighting in Vietnam, the Pro-Communist and the Anti-Communist factions. Here are some references concerning the size of both armies:

- The US involvement in Vietnam peaked in 1969 when 543,000 troops were in Vietnam territory [52].

- Another reference from the University of Illinois states that in 1969, the US involvement in Vietnam peaked with around 500,000 troops[53].
- Anthony James Joes from the Foreign Policy Research Institute, assesses that the combined US-South Vietnam strength did not achieve a 2:1 ratio over their communist counterparts [54].
- During a speech by Lyndon Baines Johnson delivered 31 March 1968, he assessed that the US and their allies were 'contributing 600,000 fighting men to assist 700,000 South Vietnamese troops in defending their little country.' This therefore estimates the combined strength as being around 1,200,000 men [55].
- Regarding the Vietcong strength in the third episode of the CIA book: 'CIA and the Vietnam Policymakers: Three Episodes 1962-1968', it is stated that 'If SD [Viet Cong Self-Defense forces] and SSD [VC Secret Self-Defense forces] are included in the overall enemy strength, the figure will total 420,000 to 431,000' [56]. They also reference that the total number of enemy troops in South Vietnam was around 600,000 [56].

Given these numbers we estimate an asymmetry of 0.33, based on assuming 1,200,000 US and allied troops (A) fighting against 600,000 pro-communist troops (B).

4.11 World War I

World War I represents a war that is rather symmetrical. Also known as The Great War, it was a conflict that was waged throughout Europe and involved Italy, Germany, the Austro-Hungarian Empire, Bulgaria/Serbia, The Ottoman Empire, Russia, France, Britain and the US. The two sides to the conflict were the Central Powers and the Entente Powers. The Central Powers were Germany, The Austro-Hungarians, Italy, The Ottoman Empire, and Bulgaria. The Entente Powers were mainly France, Russian Empire, The British Empire, Italy, and the Unites States.

4.11.1 Estimated Duration

The war is said to have begun on June 28, 1914, when Gavirilo Princip shot and killed Archduke Franz Ferdinand, heir to the Austro-Hungarian throne. When the Austro-Hungary demanded action by Serbia to punish those responsible and deemed that Serbia had not complied, it declared war. Soon after, due to protection agreements that were in place, other nations joined in the confrontation. The war ended on November 11, 1918, when Germany signed an armistice.

4.11.2 Estimated Asymmetry

We consider WWI to be a rather symmetrical confrontation with either side being able, on paper, to beat the other at one point in time during the war. The following references point to an estimated size of both sides:

- An article published in October 28, 1917 by the New York Times, stated that the Teutonic Allies had 11 million soldiers (although figures were unreliable) and the Entente Allies had 21 million men [57].
- In Arthur L. Frothingham's 'Handbook of War Facts and Peace Problems' available through the Harold B. Lee Library, the estimated strength is 20 million men for the Central Powers, and 25 million men for the Entente Powers [58] CHAPTER XI.
- The US involvement in the war was 2 million to 4 million troops [59]. However the US sent troops to Europe in 1917 after the Russian Empire surrendered.
- Mannerheim Military Academy estimates that 60 to 65 million men fought in WWI[60]
- Other estimates point that the total number of mobilized troops were 21 million for the Central Powers, and 40 million for the Entente Powers. From these, 12 million were Russians. [61] [62]
- In the book The Top Ten of Everything, the following estimates could be found: Russia 12 million, Germany 11 million, British Empire 9 million, France 8 million, Austro-Hungarian 7 million, Italy 5 million, USA 4 million [63]

Given this numbers, and the fact that the US entered after Russia had surrendered, we estimate an asymmetry of 0.11 based on the Central Powers having an average of 20 million men, and the Entente Powers having an average of 25 million men.

4.12 World War II

World War II is also rather symmetrical. It refers to the war fought between the Axis and the Allies in Europe from the late 1930's to 1945. The Axis refers to Nazi Germany and its allies. The Allies refers to Britain and its allies.

4.12.1 Estimated Duration

In 1933 Adolf Hitler came to power in Germany, and began an expansive campaign breaking and ignoring the Treaty of Versailles, which had been signed after Germany's defeat following the end of WWI. Italy invaded Ethiopia, and the Spanish Civil War began in 1936 with Germany and Spain supporting Franco. In October Germany and Italy formed the Rome-Berlin Axis, and in November Japan and Germany formed another pact. In mid 1937 Japan invaded China, and in 1938 Germany annexed Austria. Russia began to support China, and by September 1939, Germany invaded Poland. From then on the war escalated quickly. We will take September, 1939 when Germany invaded Poland, as an approximate date for the begining of the war since this is arguably when the war became a 'world war'. We will say the war ended on August 15, 1945, when Japan surrendered following the atomic bombs in Nagasaki and Hiroshima.

4.12.2 Estimated Asymmetry

Estimating the strength of the Axis army and the Allied army is very complex. The war was fought in different arenas – in Asia, Europe, and Africa. Almost all the countries of the world took some stance regarding the war. But the war in general was rather symmetrical in terms of military strength. The following references support this claim:

- The National WWII Museum in New Orleans, and The Honolulu Advertiser, estimated that the peak strengths of the different forces were 12 million US troops, 10 million German troops, 5 million French troops, 12 million Russians, 4 million British and 6 million Japanese [64] [65].
- The Grolier Multimedia Encyclopedia cited Colonel John R. Elting with the following statistics for peak strength for the Allies: USSR 12 million, UK 5 million, US 12 million, while for the Axis it was Germany 10 million, Italy 4 million, and Japan 2 million. [66]
- In the book The Top Ten of Everything, the following estimates could be found: USSR 12 million, USA 12 million, Germany 10 million, Japan 6 million, France 5 million, UK 4.5 million, Italy 4.5 million, China 3 million, Poland 1 million. [63]

Before we estimate some measure of asymmetry, we should take into account that the war was fought on a large scale, from Asia to Europe. We also take into account that the US and USSR had to divide their total troop strengths between support for China, and support for the UK. Also, France surrendered in 1940 and the UK engaged the French navy in order to prevent it from being captured in the war. This was also the case for Poland. Given the rough numbers and the above issues, we estimate the asymmetry between the two factions on the European front to be 0.01 in favor of the Axis, and 0.1 in favor of the Allies on the Asian front. We will therefore take an overall asymmetry of 0.05. In fact, WWII acts as an approximate 'zero' in terms of measuring the asymmetry for other conflicts, since either side at their peak strength could have won in WWII.

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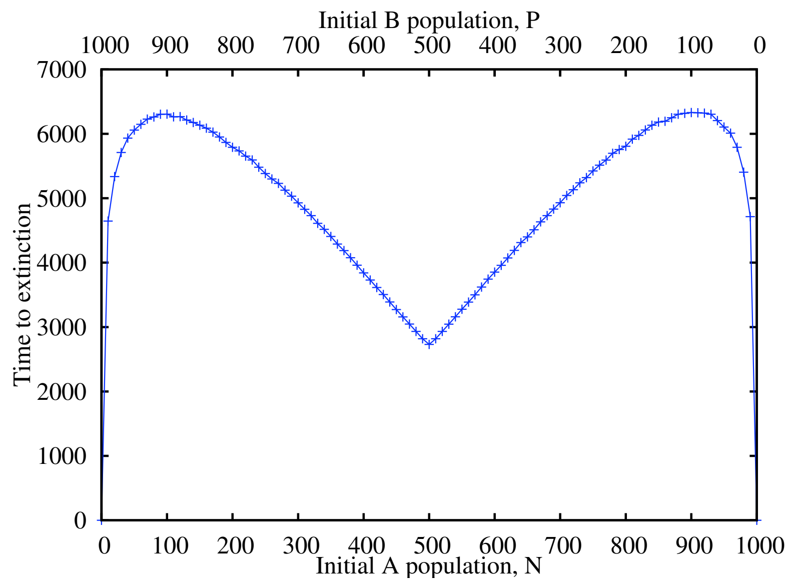

Figure FI 7: Time to extinction dependence on initial populations in EFF model. Initial total population $N + P = 1000$.

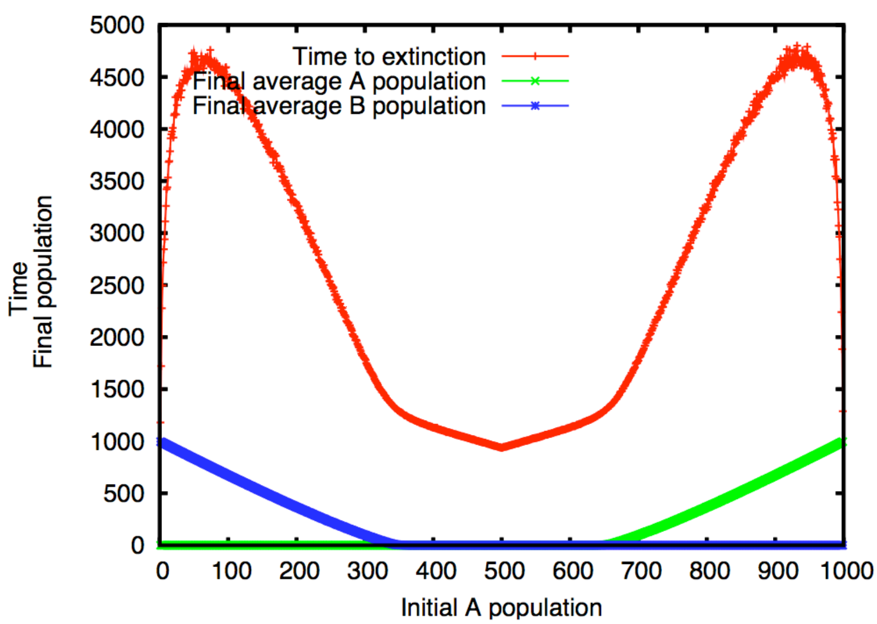

Figure FI 8: Time to extinction (T) for the minority advantage variant of our model, as a function of the initial A population (N). Here the total population $N + P = 1000$.

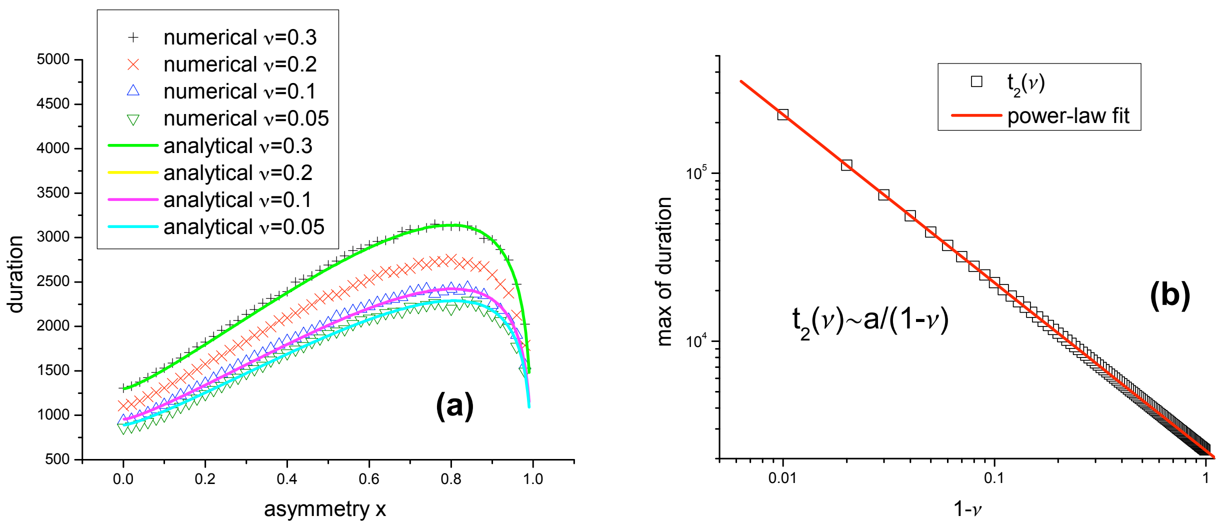

Figure FI 9: (a) Duration as a function of asymmetry. Total population $N = 1000$ is constant, and the numerical simulation is a run-average over 1000 runs. (b) Maximum duration as a function of $1-\nu$. Here $x = x_{max} = 0.788$ and $t_2(\nu) = T(x_{max}, \nu)/t_1(x_{max}) \simeq 2228.977/(1-\nu)$